AD-A049 548 MINNESOTA UNIV MINNEAPOLIS DEPT OF THEORETICAL STATISTICS F/G 12/1
SIMULATION STUDIES ON SOME NEAREST NEIGHBOR RULES FOR STATISTIC--ETC(U)
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UNCLASSIFIED UMN/DTS/TR-303 ARO-13149.4-M NL

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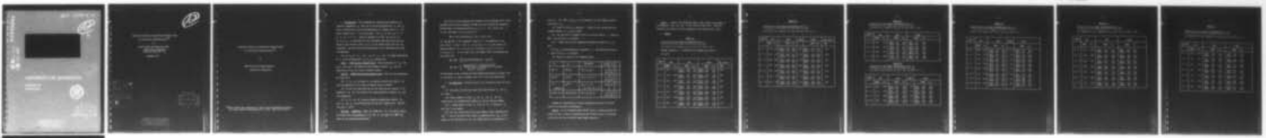

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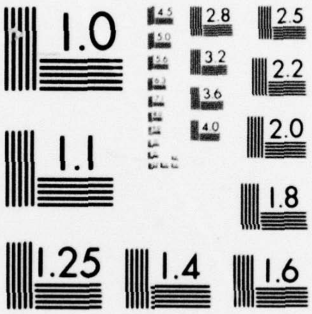
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12

Simulation Studies on Some Nearest Neighbor Rules for Statistical Classification.(1)

By

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University of Minnesota

Technical Report No. 303

November 1977

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Simulation Studies on Some Nearest Neighbor Rules for Statistical Classification.[1]

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David Aarons and Somesh DasGupta

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[1] This research was supported by a grant from the Mathematics Division, U.S. Army, Research TrianglePark, N.C.; Grant DAAG 29-76-G-0038.

1. Introduction. The two-population classification problem is to identify a population π_0 with one of two given populations π_1 and π_2 based on observations from these populations on a random vector X. We shall consider here X to be univariate. Let F_i be the c.d.f. of X in π_i $(i = 0, 1, 2)$. Thus our problem is to test $H_1: F_0 = F_1$ vs. $H_2: F_0 = F_2$. In this paper we have considered some rules which are suggested in the literature when F_1, F_2 are not known except that they are continuous. We have studied the performances of the following three rules by simulation.

Let X_0, X_{1i} $(i = 1,\dots,n_1)$, X_{2i} $(i = 1,\dots,n_2)$ be random observations on X from the populations π_0, π_1, π_2, respectively.

Rule I. 1-NN (nearest neighbor) Rule: Measure distances of X_0 from X_{1i}'s and X_{2i}'s and based on these distances classify X_0 into the population to which its nearest neighbor belongs.

Rule II. 1-RNN (rank nearest neighbor) Rule: Pool all the observations and order them.

(a) If X_0 is the largest or the smallest observation classify X_0 into the population of its nearest neighbor (based on ranks).

(b) If both the right-hand and the left-hand nearest neighbor of X_0 (denoted by U_1 and V_1) belong to the same population, classify X_0 into that population.

(c) If U_1 and V_1 belong to different populations classify X_0 into π_1 and π_2 with probabilities 1/2 and 1/2, respectively. (We call this case a "tie".)

Rule III. 2-RNN Rule: Apply the 1-RNN rule. If a tie occurs, delete the observations corresponding to U_1 and V_1 and apply the 1-RNN rule again on the remaining observations.

The first rule was suggested and studied by Fix and Hodges (1951, 1953). DasGupta and Lin (1977) proposed the RNN rules and obtained the asymptotic probabilities of misclassification as $n_1, n_2 \to \infty$. For a given rule δ, let its PMC under $F_0 = F_1$ be given by

$$\alpha(\delta) = \Pr[\delta \text{ classifies } X_0 \text{ into } \pi_2 \,|F_0 = F_1] .$$

Let $\alpha_1^*, \alpha_2^*, \alpha_3^*$ be the asymptotic values of α corresponding to the above rules 1, 2 and 3. Let f_i be the p.d.f. of F_i with respect to Lebesgue measure $(i = 1,2)$ and $p_i = \lim n_i/(n_1 + n_2)$ $(i = 1,2)$ as $\min(n_1,n_2) \to \infty$. It was shown by Fix and Hodges (1951) and DasGupta and Lin (1977) that

$$\alpha_1^* = \alpha_2^* = \int_{-\infty}^{\infty} p_2 f_1(x) f_2(x) dx / \{p_1 f_1(x) + p_2 f_2(x)\}$$

$$\alpha_3^* = \alpha_2^* + \int_{-\infty}^{\infty} \frac{p_1 p_2 f_1(x) f_2(x) \cdot \{p_2 f_2(x) - p_1 f_1(x)\}}{\{p_1 f_1(x) + p_2 f_2(x)\}^3} f_1(x) dx .$$

In this paper we have studied the finite-sample performances of these rules by estimating α based on samples from sets of two given populations.

2. <u>The Experiment</u>. Different steps of our simulation study are given below.

(i) Two known but different univariate distributions F_1 and F_2 are chosen.

(ii) Random samples of sizes n_1 and n_2 from F_1 and F_2 , respectively, are obtained; these samples are called training samples.

(iii) A random sample of size n_0 from $F_0 = F_1$ is obtained. We call this a test sample.

(iv) For each observation in the test sample a given classification rule δ (one of the above three rules) is applied and let n_{02} be the number of the observations in the test sample which are classified by δ

into F_2 . Let $\hat{\alpha}(\delta) = n_{02}/n_0$ be the proportion of test samples misclassified into F_2 .

(v) Steps (ii)-(iv) are repeated r times for new training and test samples keeping n_1, n_2 and n_0 fixed.

(vi) The mean and the standard error of the mean based on r values of $\hat{\alpha}(\delta)$ thus obtained are recorded.

(vii) Steps (ii)-(vi) are repeated for different values of n_1, n_2 and r .

(viii) F_2 is characterized by a parameter θ . For different values of θ steps (i)-(vii) are repeated.

Our choices are given in the following table.

F_1	F_2	Parameters	$n_1=n_2$	n_0	r
N(0,1)	N(θ,1)	θ=0, ±1, ±2, 3	25 100	100 400	20 4
N(0,1)	N(0,θ)	θ=2, 3, 1/2, 1/3	25 100	100 400	20 4
e^{-x} (density)	$\theta e^{-\theta x}$	θ=1, 2, 3, 4, 1/2, 1/3, 1/4, 1/8	100	100	20
Cauchy (0,1)	Cauchy (θ,1)	θ=0, ±1, ±2, ±3	25 100	100 400	20 4

Samples are generated by a library subroutine available on the CDC 6400 at the University of Minnesota.

Note 1. In the following tables "Half" refers to taking one-half the number of ties to count as misclassified and "R-half" refers to resolving the ties by the use of uniform random number generator.

Note 2. In some of the following tables EPMC denotes an estimate of the asymptotic PMC ($\alpha_1^* = \alpha_2^*$) of the 1-NN and 1-RNN rules. These are derived by the method of runs as suggested in Das Gupta and Lin (1977).

3. Tables

Table 3.1

Proportion of test sample misclassified into π_2.

$F_1 = N(0,1)$, $F_2 = N(\theta,1)$; $n_1 = n_2 = 25$, $n_0 = 100$, $r = 20$.

Optimal (assuming θ is known and for minimax rule) PMC is

$\Phi(-|\theta|/2)$.

θ \ Rule	1NN		RNN			2-RNN			Opt. Exp't.
	MEAN	s.e.		MEAN	s.e.		MEAN	s.e.	PMC
$\theta = 0$	.479	.017	Half Rhalf	.479 .485	.013 .016	Half Rhalf	.479 .484	.014 .015	.500
$\theta = 1$	.374	.018	Half Rhalf	.381 .374	.014 .016	Half Rhalf	.343 .340	.021 .021	.308
$\theta = -1$	.426	.020	Half Rhalf	.426 .432	.014 .017	Half Rhalf	.421 .425	.025 .024	.308
$\theta = 2$	.195	.018	Half Rhalf	.194 .196	.018 .018	Half Rhalf	.165 .164	.017 .018	.159
$\theta = -2$	.245	.020	Half Rhalf	.254 .251	.018 .018	Half Rhalf	.258 .255	.019 .018	.159
$\theta = 3$	.086	.012	Half Rhalf	.089 .084	.012 .011	Half Rhalf	.062 .061	.010 .009	.067
$\theta = -3$	.105	.013	Half Rhalf	.114 .113	.012 .011	Half Rhalf	.119 .118	.015 .015	.067

Table 3.2

Proportion of test sample misclassified into π_2 .

$F_1 = N(0,1)$, $F_2 = N(\theta,1)$; $n_1 = n_2 = 100$, $n_0 = 400$, $r = 4$.

θ \ Rule	1NN		RNN			EPMC	2-RNN			Opt. Exp't.
	MEAN	s.d.		MEAN	s.d.			MEAN	s.d.	PMC
θ = 0	.490	.018	Half Rhalf	.482 .475	.008 .006	.48	Half Rhalf	.509 .501	.014 .016	.500
θ = 1	.415	.010	Half Rhalf	.398 .404	.014 .024	.36	Half Rhalf	.351 .358	.009 .024	.308
θ = -1	.402	.010	Half Rhalf	.394 .397	.007 .007	.38	Half Rhalf	.347 .344	.025 .024	.308
θ = 2	.208	.010	Half Rhalf	.210 .208	.010 .009	.22	Half Rhalf	.200 .199	.011 .012	.159
θ = -2	.209	.012	Half Rhalf	.213 .215	.008 .009	.22	Half Rhalf	.197 .200	.013 .014	.159
θ = 3	.088	.011	Half Rhalf	.083 .082	.009 .007	.10	Half Rhalf	.065 .066	.005 .006	.007
θ = -3	.104	.012	Half Rhalf	.101 .107	.008 .013	.09	Half Rhalf	.088 .094	.012 .014	.007

Table 3.3.

Proportion of test sample misclassified into π_2 .

F_1 = N(0,1), F_2 = N(0,θ); n_1 = n_2 = 25, n_0 = 100. r = 20 .

θ \ Rule	1NN		RNN			2-RNN		
	MEAN	s.e.		MEAN	s.e.		MEAN	s.e.
θ = 2.0	.375	.009	Half	.394	.008	Half	.353	.014
			Rhalf	.393	.010	Rhalf	.355	.015
θ = 3.0	.399	.014	Half	.346	.013	Half	.293	.019
			Rhalf	.337	.013	Rhalf	.295	.018
θ = .5	.417	.017	Half	.438	.015	Half	.461	.020
			Rhalf	.337	.018	Rhalf	.460	.021
θ - 1/3	.359	.022	Half	.376	.018	Half	.393	.019
			Rhalf	.380	.019	Rhalf	.391	.019

Table 3.4

Proportion of test sample misclassified into π_2 .

F_1 = N(0,1), F_2 = N(0,θ); n_1 = n_2 = 100, n_0 = 400, r = 4 .

θ \ Rule	1NN		RNN			EPMC	2-RNN		
	MEAN	s.e.		MEAN	s.e.			MEAN	s.e.
θ = 2.0	.435	.022	Half	.424	.022	.36	Half	.395	.027
			Rhalf	.426	.025		Rhalf	.396	.028
θ = 3.0	.333	.012	Half	.338	.010	.32	Half	.295	.012
			Rhalf	.336	.011		Rhalf	..296	.011
θ = .5	.397	.062	Half	.405	.011	.38	Half	.409	.006
			Rhalf	.407	.013		Rhalf	.408	.005
θ = 1/3	.339	.021	Half	.352	.020	.35	Half	.360	.029
			Rhalf	.354	.021		Rhalf	.361	.030

Table 3.5

Proportion of test sample misclassified into π_2 .

$f_1(x) = e^{-x}$, $f_2(x) = \theta e^{-\theta x}$; $n_1 = n_2 = n_0 = 100$, $r = 4$.

θ \ Rule	1NN		RNN			EPMC	2-RNN		
	MEAN	s.e.		MEAN	s.e.			MEAN	s.e.
θ = 1	.508	.016	Half	.509	.013	.47	Half	.503	.013
			Rhalf	.523	.016		Rhalf	.517	.015
θ = 2	.442	.015	Half	.434	.014	.38	Half	.442	.016
			Rhalf	.438	.017		Rhalf	.444	.016
θ = 3	.402	.014	Half	.388	.011	.36	Half	.394	.013
			Rhalf	.387	.011		Rhalf	.387	.014
θ = 4	.335	.009	Half	.330	.007	.32	Half	.327	.009
			Rhalf	.336	.008		Rhalf	.330	.009
θ = .5	.453	.010	Half	.453	.009	.38	Half	.430	.010
			Rhalf	.458	.013		Rhalf	.430	.014
θ = 1/3	.410	.011	Half	.395	.008	.36	Half	.346	.010
			Rhalf	.386	.009		Rhalf	.335	.010
θ = 1/4	.354	.015	Half	.364	.012	.32	Half	.290	.013
			Rhalf	.372	.013		Rhalf	.292	.013
θ = 1/8	.247	.014	Half	.248	.012	.22	Half	.181	.011
			Rhalf	.259	.014		Rhalf	.185	.010

Table 3.6

Proportion of test sample misclassified into π_2 .

F_1 = Cauchy(0,1), F_2 = Cauchy($\dot{\theta}$,1); $n_1 = n_2 = 25$, $n_0 = 100$, $r = 20$.

θ \ Rule	1NN		RNN			2-RNN		
	MEAN	s.e.		MEAN	s.e.		MEAN	s.e.
θ = 0	.473	.018	Half	.430	.015	Half	.488	.027
			Rhalf	.493	.018	Rhalf	.505	.029
θ = 1	.406	.022	Half	.418	.022	Half	.397	.031
			Rhalf	.408	.025	Rhalf	.395	.033
θ = -1	.398	.016	Half	.410	.012	Half	.389	.021
			Rhalf	.410	.013	Rhalf	.385	.022
θ = 2	.288	.021	Half	.297	.021	Half	248	.027
			Rhalf	.288	.021	Rhalf	.238	.028
θ = -2	.247	.012	Half	.264	.012	Half	.248	.017
			Rhalf	.276	.015	Rhalf	.252	.019
θ = 3	.161	.020	Half	.168	.017	Half	.103	.017
			Rhalf	.161	.018	Rhalf	.099	.017
θ = -3	.153	.015	Half	.156	.013	Half	.130	.014
			Rhalf	.154	.013	Rhalf	.125	.014

Table 3.7

Proportion of test sample misclassified into π_2 .

F_1 = Cauchy(0,1), F_2 = Cauchy(θ,1); $n_1 = n_2 = 100$, $n_0 = 400$, $r = 4$.

θ \ Rule	1NN		RNN			2-RNN		
	MEAN	s.e.		MEAN	s.e.		MEAN	s.e.
θ = 0	.494	.015	Half	.514	.013	Half	.506	.017
			Rhalf	.529	.014	Rhalf	.512	.021
θ = 1	.411	.010	Half	.426	.009	Half	.381	.018
			Rhalf	.446	.018	Rhalf	.390	.017
θ = -1	.457	.029	Half	.446	.033	Half	.394	.028
			Rhalf	.454	.025	Rhalf	.393	.025
θ = 2	.284	.007	Half	.278	.008	Half	.217	.033
			Rhalf	.283	.009	Rhalf	.219	.024
θ = -2	.152	.016	Half	.318	.022	Half	.254	.014
			Rhalf	.321	.015	Rhalf	.257	.010
θ = 3	.152	.016	Half	.154	.015	Half	.088	.018
			Rhalf	.417	.012	Rhalf	.087	.014
θ = -3	.204	.034	Half	.199	.032	Half	.105	.011
			Rhalf	.198	.034	Rhalf	.103	.012

4. Concluding Remarks. For all the three rules considered, it seems that $\hat{\alpha}_1$ has a definite tendency to decrease as θ moves away (in either direction) from its value under F_1.

For small $n_1 = n_2$ there is not any marked difference in performances of these three rules although the 2-RNN rule may be a bit better. However, for large $n_1 = n_2$ the 2-RNN rule seem to have markedly better performance except for the cases $N(0,1)$ vs. $N(0,\theta)$, $\theta < 1$. This report is the first empirical study on the performances of 1NN and RNN rules, although a more detailed study especially on multi-stage RNN rules is called for.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER 14 UMN/DTS/TR-303
Technical Report No. 303

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 Simulation Studies on Some Nearest Neighbor Rules for Statistical Classification,

5. TYPE OF REPORT & PERIOD COVERED
9 Technical Report,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
10 Somesh/Das Gupta ~~and~~ David/Aarons

8. CONTRACT OR GRANT NUMBER(s)
15 ✓ DAAG-29-76-G-~~0~~038 ✓

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Department of Theoretical Statistics ✓
University of Minnestoa
Minneapolis, MN 55455

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709

12. REPORT DATE
11 Nov[redacted]77

13. NUMBER OF PAGES
11 12 15 p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
18 ARO 19 13149.4-M

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
NA

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Classification, nearest neighbor rule, rank nearest nearest neighbor rule, simulation, probability of misclassification.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Abstract: The performances of the distance nearest neighbor rule, one-stage rank nearest neighbor rule and two-stage rank nearest neighbor rule for the two-population statistical classification problem are studied through simulatión processes. Normal distributions with different mean, normal distributionswith different variances, exponential distributions and Cauchy distributions are considered.

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